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# WORM GEAR ANALYSIS IN TERMS OF NUMERICAL MODELLING

*SUMMARY*

*A worm gear mechanism, widely used in numerous mechanical applications, is the essential component in energy efficiency considerations. Compared to traditional gearboxes, worm gear is characterized by lower value of efficiency, provided by the manufacturers as 70–90%. However, the efficiency level can differ depending on the specific conditions of operation such as output torque, input rotational velocity, ambient temperature and type of lubrication. In order to analyse and simulate the worm gear mechanism in mechanical systems, it is essential to establish the actual efficiency value for specific conditions. The following article contains the description of efficiency test performed for the worm gear as a part of tram pantograph system studied in frame of ESTOMAD project. The aim of the experiment was to determine the actual efficiency value of the transmission tested. The worm gear has been tested in an open loop configuration, therefore the efficiency has been determined basing on input and output torque and rotational velocity measurements. The tests have been performed for various output torques and input velocities to obtain view on losses in worm gear under different operational conditions, paying special attention to the velocity that drives the pantograph mechanism. The efficiency changes with the output torque variation, while the impact of rotational velocity level on the mentioned value is minimal. The results of measurements are used in modelling phase of worm gear component in 1D simulation environment, particularly creating new model and its correlation and validation.*

***Keywords****: worm gear, pantograph mechanism, efficiency, open loop configuration, 1D model*

*ANALIZA SPRAWNOŚCI PRZEKŁADNI ŚLIMAKOWEJ NORD FLEXBLOC SK 1SI63*

*Mechanizm przekładni ślimakowej, szeroko stosowany w wielu urządzeniach mechanicznych, jest elementem charakteryzującym się niską sprawnością, co nie jest dobrze postrzegane z energetycznego punktu widzenia. W porównaniu z tradycyjnymi przekładniami, przekładnia ślimakowa charakteryzuje się niższą wartością sprawności, określanej przez producentów na poziomie 70–90%. Jednakże poziom sprawności takiej przekładni może zmieniać się w zależności od warunków pracy, takich jak obciążenie, wejściowa prędkość obrotowa, temperatura otoczenia oraz sposób smarowania. W celu analizy i symulacji działania przekładni ślimakowej stosowanej w konkretnym systemie mechanicznym, ważne jest, aby określić właściwy poziom sprawności w określonych warunkach pracy układu. Artykuł ten przedstawia badania sprawności przekładni ślimakowej, będącej częścią pantografu tramwajowego analizowanego w ramach projektu ESTOMAD. Celem eksperymentu było określenie rzeczywistej wartości sprawności badanej przekładni. Przekładnia ślimakowa została poddana testom na stanowisku zbudowanym w postaci otwartego łańcucha kinematycznego, dlatego też sprawność została określona na podstawie pomiarów momentu oraz prędkości obrotowej na wejściu i wyjściu przekładni. Testy były przeprowadzone dla różnych obciążeń i prędkości obrotowych, aby oszacować straty w urządzeniu w zróżnicowanych warunkach eksploatacyjnych, w szczególności biorąc pod uwagę prędkości obrotowe stosowane w przypadku mechanizmu podnoszenia pantografu. Sprawność przekładni zmienia się wraz ze zmianą momentu obciążającego, natomiast wpływ zmian prędkości obrotowej na sprawność był bardzo niewielki. Wyniki pomiarów są wykorzystane w procesie modelowania przekładni w środowisku symulacyjnym 1D, podczas tworzenia nowego modelu oraz jego korelacji i walidacji.*

**Słowa kluczowe***: przekładnia ślimakowa, pantograf, sprawność, konfiguracja „open loop”, model 1D*

## 1. INTRODUCTION

The aim of the experiments presented in the following article was to determine the losses in the worm gear for different operating conditions such as torque on the output and the influence of rotational velocity on the input of transmission. The worm gear is a part of the tram pantograph system analysed in frame of the ESTOMAD project in the context of overall efficiency of the mechanism. Worm gear is a part of pantograph drive subsystem along with a DC motor and a lead screw. The input torque is provided to the worm by the electric motor, while the worm wheel shaft is connected to the lead screw. The complete drive ensures the up and down motion of pantograph mechanism to connect the tram with electric line. Worm gear was identified as an element contributing to the greatest extent of losses in the drive. Losses in transmission are the cause of elevated temperature and significant load of the electric motor, which could lead to damage and disable the drive in long term tests. Therefore, a detailed study of mechanism efficiency was necessary.

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The transmission tests have determined the overall efficiency level of the device for different values of output torque and input rotational velocity. Additional motivation to conduct this type of experiment was to create a numerical model of the worm gear in order to perform the simulations of this component in 1D environment. In this case it was necessary to determine the level of different types of losses, both in tooth engagement of worm and worm wheel, as well as resulting from the shafts bearing and oil churning and build proper mathematical model of these losses.

## 2. LABORATORY TEST OF WORM GEAR

This chapter is devoted to laboratory tests of worm gear. In first subchapter object of tests, namely gearbox NORD FLEXBLOC SK 1SI63 is presented with its technical data (Flexbloc & Minicase Worm Drivers). Second subchapter describes test rig that has been used for determination of energetic parameters of investigated object. Another subchapter briefly describes scope of measurements, which have been conducted. Last subchapter presents obtained results along with simple conclusions.

### 2.1. Technical specification

The worm gear analysed in frame of the article is a typical transmission consisted of worm and worm wheel. The particular tested gearbox is shown in the Figure 1.

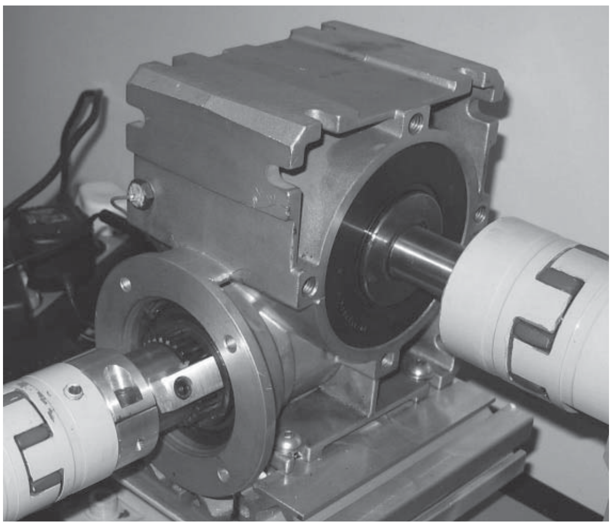

**Fig. 1.** Object of testing

A short technical specification has been presented in Table 1. Despite the fact that efficiency value has been provided, it was necessary to determine the actual efficiency levels for specific working conditions of variable torque, rotational velocity and lubrication.

**Table 1**

Essential parameters of tested worm gear

| Parameter | Value |
|---|---|
| Nominal efficiency (at $n_1$ = 1400 min$^{-1}$) | 89% |
| Start-up efficiency | 67% |
| Gear ratio | 7.5 |
| Worm wheel diameter | 90 mm |
| Worm diameter | 63 mm |
| Normal pressure angle | 20° |
| Lubricant | CLP PG 220 (Alphasyn GS 220) |

The analysis of device required thorough knowledge about geometrical parameters describing the engagement between worm and worm wheel. Basic geometrical parameters used for calculations have been summarized in Table 2.

**Table 2**

Basic geometrical parameters

| Parameter | Worm | Worm wheel |
|---|---|---|
| Teeth number [mm] | 4 | 30 |
| Measured diameter [mm] | 42.2 | 97.7 |
| Width [mm] | 39.4 | 25 |
| Gear ratio | 7.5 | |

The quantities crucial for further investigation have been calculated based on parameters from Table 2 and presented in Table 3.

**Table 3**

Calculated geometrical parameters

| Parameter | Worm | Worm wheel |
|---|---|---|
| Lead angle $\lambda$ [°] | 18.92 | 71.08 |
| Helix angle $\psi$ [°] | 71.08 | 18.92 |
| Apparent friction angle Φn [°] | 6.73 | 19 |
| Pitch diameter d [mm] | 35 | 90 |

Shaft's bearing, as a component contributing to the overall losses, has been realized with use of standard ball bearing codenamed as shown in Table 4.

**Table 4**

Ball bearings models used in analysed gearbox

| Bearing model | |
|---|---|
| 6211 C 43,0 (worm wheel shaft) | 2 pcs (worm wheel shaft) |
| 6009 2Z C3 | 1 pc (worm shaft) |
| 6202 | 1 pc (worm shaft) |

Gearbox lubrication is ensured by the oil circulating in the housing in the amount of 180 ml. Parameters of used lubricant are quite important in terms of energy loss. The manufacturer provides a worm gear with Alphasyn GS 220 (CLP PG type) oil with parameters summarized in Table 5 (Datasheet of GS 220 oil). The analysis of losses requires the knowledge about the oil viscosity characteristic in a range of operating temperature. The investigation is presented in the further chapters of the following paper.

**Table 5**

Specification of CLP PG 220 oil type

| Parameter | Value |
|---|---|
| Density | 1 g/ml |
| Kinematic viscosity (at 40°C) | 220 cSt |
| Kinematic viscosity (at 100°C) | 36 cSt |

### 2.2. Measurement setup

The entire experimental campaign was performed in an open loop configuration test rig. In such configuration the energy is transferred from the electric motor to the powder brake through the worm gear and the system of couplings. Within the test the following quantities have been acquired: rotational velocity on the input shaft (obtained directly from the VFD controller), torque on the input and output shaft, oil temperature in the worm gear housing. Rotational velocity on the output shaft has been calculated based on the gear ratio. The diagram of the test rig is shown in the Figure 2.

The test rig has been driven with a 4kW three-phase induction motor with rated speed of 2815 rpm, manufactured by TAMEL. The motor has been controlled with LG iS5 frequency converter (inverter). The inverter allows scalar control with U/f linear characteristic and what is more sensor and sensorless vector control of torque or rotational velocity. During the tests, the inverter was working in vector control mode, which ensured the operation with the rotational velocity on the established level. The tested worm gear has been loaded with powder brake with power dissipation of 2 kW, manufactured by EMA-ELFA, type P170HV. The brake ensures the 5% accuracy maintaining the value of torque in the whole range of rotational velocity. The brake's output torque is independent of slip speed and working temperature. MECMESIN torque sensors have been installed on the input and output of worm gear. The signals from torque sensors and the rotational velocity have been acquired with SPIDER-8 measurement system manufactured by HOTTINGER. Temperature measurement is based on the resistance temperature detector PT100 installed in the worm gear housing. The resistance of the detector has been measured with METEX multimeter. The connections of all shafts of drive and load systems of test rig have been executed with torsionally flexible couplings, ROTEX (KTR) type. The drive from the motor to the gearbox input shaft is transmitted via a toothed belt. In order to decrease the influence of external conditions the ambient temperature has been as stable as possible during the tests (22°C). The test rig described in the paper has been built specially for this particular worm gear tests. The complete setup, except data acquisition instruments is shown in Figure 3.

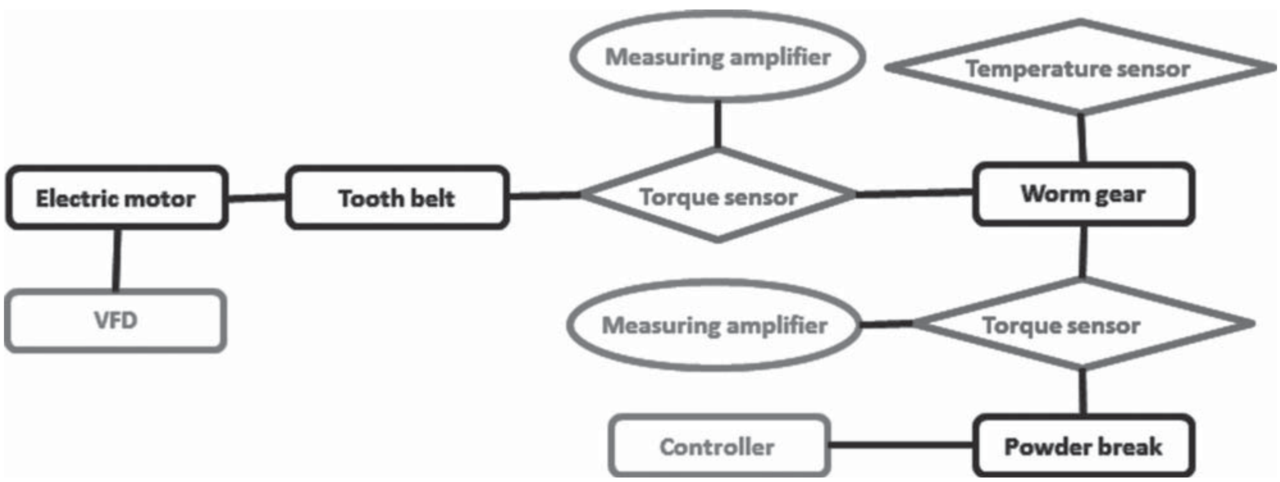


**Fig. 2.** Test rig configuration

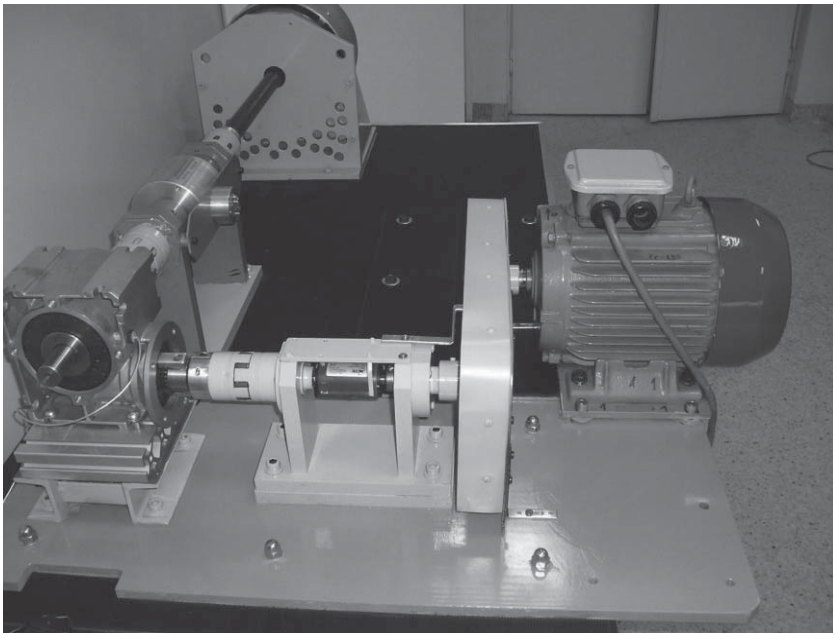

**Fig. 3.** General view of test rig used in the experiments

### 2.3. Test plan

Tests have been performed for various operational conditions in order to provide the essential information concerning efficiency characteristic of analysed transmission. The test cases have been summarized in Table 6.

The efficiency has been determined based on the output to the input power ratio calculated with use of input and output torque and input velocity data.

Along with torque and rotational velocity, the temperature of oil has been acquired during every individual test in order to investigate the influence of the temperature on the overall efficiency of tested transmission.

Large number of test configurations allowed to investigate the influence of individual parameters on the overall worm gear efficiency. Tests performed for constant output torque provided information about the impact of the other parameters (rotational velocity, temperature) on the efficiency value. Analogically, experiment with fixed temperature has been done. The results of tests without torque on the output provided the information about the churning losses characteristic, since bearing and tooth engagement losses are extremely small during the non-load test and have been ignored. Therefore, it was possible to prepare relevant experimental model of churning power losses. Additionally, the experiments with standard and decreased oil level have been executed in order to check the impact of lubrication on the overall energetic behaviour of device.

### 2.4. Experimental results

The general view on losses during the work in various operational conditions is presented in Figure 4 as efficiency characteristics in function of output torque vs. input rotational velocities.

As mentioned in previous chapter, the tests have been performed for various configurations of operational parameters to obtain good view on losses under different operational conditions. Figure 5 shows the efficiency characteristic as a function of output torque acquired at 1500 and 2000 rpm. During this particular experiment the oil temperature has been stable and equal to 58°C.

Based on the presented results, it can be stated that overall efficiency of the device is mainly load dependant. Increased temperature and therefore lower viscosity of oil contributed to efficiency rise of about 3%. The influence of rotational velocity on the efficiency is also minimal.

Aside from measurements done in working conditions, measurements have been also conducted in idle condition. Results are presented in Figure 6.

**Table 6**

Test cases used in worm gear experiments

| Input rotational velocity [rpm] | Output torque [Nm] | Lubrication |
|---|---|---|
| 1200, 1500, 2000,2400, 2800 | No torque, 5, 15, 25, 30, 40, 50, 60, 70, 80, 90 | Standard oil level, decreased oil level |

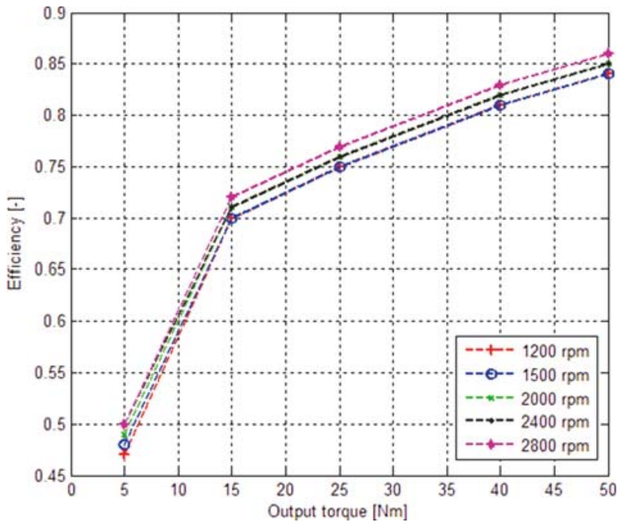


**Fig. 4.** Efficiency characteristics of tested worm gear

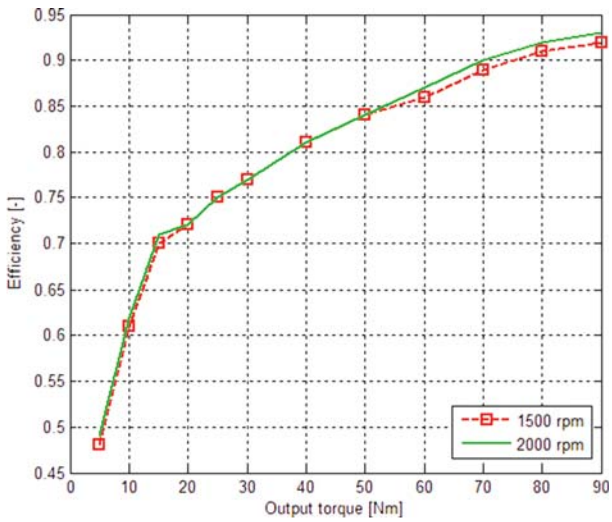


**Fig. 5.** Efficiency as a function of output torque for 1500 and 2000 rpm (at constant temperature = 58°C)

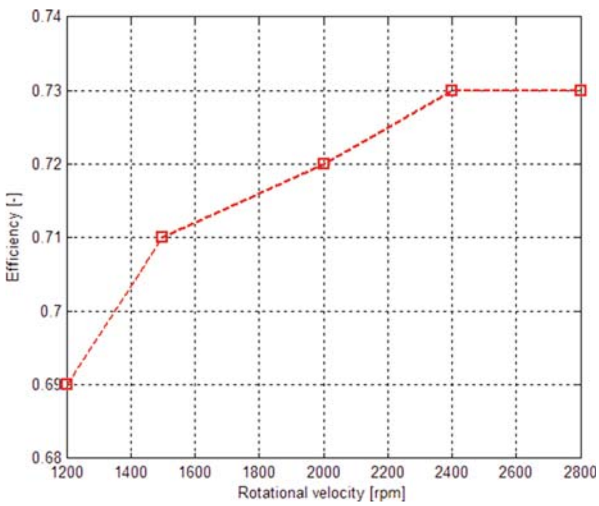


**Figure 6.** Efficiency as a function of rotational velocity (idle condition)

## 3. MATHEMATICAL MODEL OF THE ENERGY LOSS

This chapter is devoted to mathematical modelling of worm gear along with model building, correlation and validation process. At the beginning considerations concerning origin of losses are presented. Another three subchapters presents mathematical models for particular type of losses investigated. In case of tooth engagement losses and churning losses, basic formulas are given. As for bearing losses, they are presented with geometrical constrains. Gearbox geometry has been measured and presented in subchapter 3.4. This geometry has been used for determination of forces on particular bearings. Thanks to well established mathematical models of bearings power losses as a function of force it was possible to determine, bearing power losses as a function of input torque and friction coefficient. Last two subchapters considered mathematical model correlation and validation.

### 3.1. Origin of losses

Understanding the origin of losses is very important aspect of efficiency optimizing process. Having this knowledge one can determine influence of operating parameters on total losses of the system and indicate most important source of those losses. Losses in worm gear can be divided into three main types: tooth engagement losses, oil churning losses and bearing losses. The overall power loss is equal to the sum of mentioned components. The analysis of mathematical models available in the literature along with experiments performed, results in creation of complete model of overall power loss in the worm gear, ready to be implemented to the 1D simulation environment.

### 3.2. Losses due to tooth engagement

The literature provide relevant model of tooth engagement losses in the transmission (Gopinath and Mayuram). These losses are represented by the equation:

$$L_t = 2,3\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)W \tag{1}$$

where:
$L_t$ – tooth engagement power losses [kW]
$\mu$ – coefficient of friction between the teeth [–]
$Z_1$, $Z_2$ – numbers of teeth on worm and gear [–]
$W$ – total power transmitted [kW]

### 3.3. Churning losses

Churning losses model have been built as an empirical model with use of data obtained during zero torque measurements. The model is mainly based on the empirical models from available literature, as there is no well-established mathematical model for churning losses, just general formula has been used, and coefficients have been assumed to be unknown and to be correlated (Stavytskyy et al. 2010). Proper model of churning losses require the knowledge about the oil viscosity characteristic. Viscosity of oil is temperature dependent and with growing temperature viscosity decreases according to Arrhenius model (Connors 1990). Kinematic viscosity chaaracteristic can be determined with the following equation:

$$\mu = \mu_0 \cdot e^{-bT} \tag{2}$$

where:
$T$ – operating temperature
$\mu_0$ – linear coefficient
$b$ – exponential coefficient

The values of coefficients $\mu_0$ and b have been obtained with use of two kinematic viscosity values for fixed temperatures presented in previous chapters (Table 5). The characteristic of oil viscosity has been plotted and shown in Figure 7. Please note that Y axis is expressed in logarithmic scale.

For churning losses determination it was assumed that churning losses ($L_{CH}$) are non-load-dependent while bearing losses ($L_B$) and tooth engagement losses ($L_T$) are load dependent. Therefore, measurement data used in churning losses modelling has been acquired with no load on the output shaft of the worm gear. Basing on the initial assumptions, it can be stated that $L_B = 0$ and $L_T = 0$. As a result, churning loss value is equal to the overall loss. Subsequently, power loss and viscosity values have been calculated with use of measurement data. Finally, the complete empirical model of churning losses as a function of rotational velocity and oil temperature has been built

$$L_{CH} = \mu(T)(\alpha \cdot \omega^{\beta} + C) \tag{3}$$

where:
$\alpha$ – linear coefficient
$\beta$ – exponential coefficient
$C$ – constant
$\omega$ – rotational velocity

### 3.4. Losses due to friction in bearings

Bearing losses consideration required the knowledge about the forces distribution present in every bearing in the worm gear. First step was to analyse the forces acting in the worm and the gear. The forces acting on the worm are distributed as shown in Figure 8. The forces acting on worm wheel are analogical to the forces on worm.

The forces acting in gear's contact point can be transmitted to the forces in bearings (C and D) with use of torque equations about selected axis and through selected points. The schematic drawing shown on Figure 9 shows forces distribution and crucial distances used for equations building.

Analogical approach has been adapted to worm wheel shaft analysis. Similar schematic drawing has been prepared as shown in Figure 10.

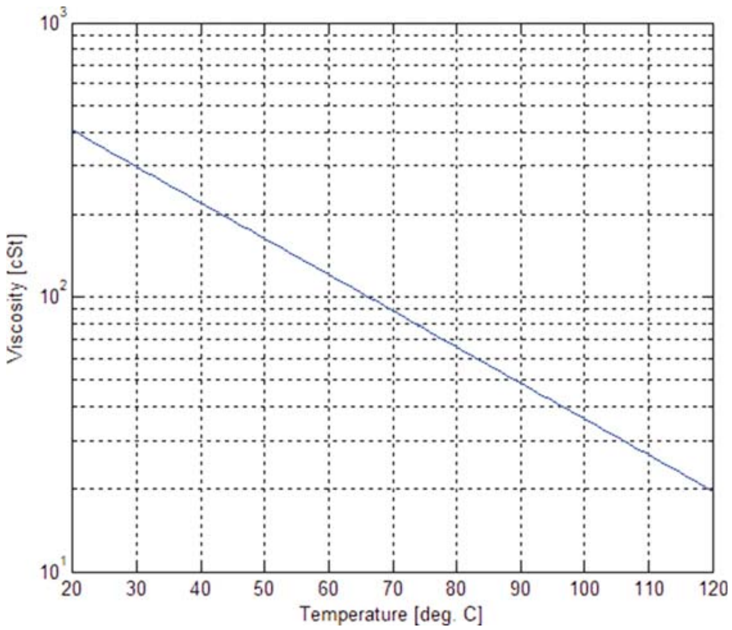


**Fig. 7.** Viscosity of oil characteristic in range of temperature

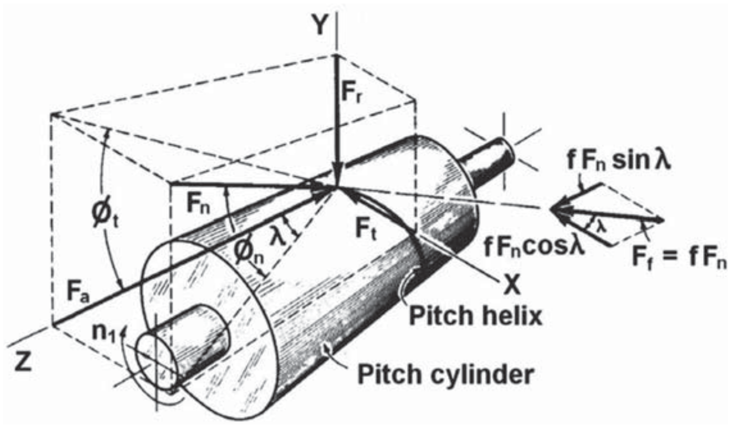


**Fig. 8.** Forces distribution on the worm

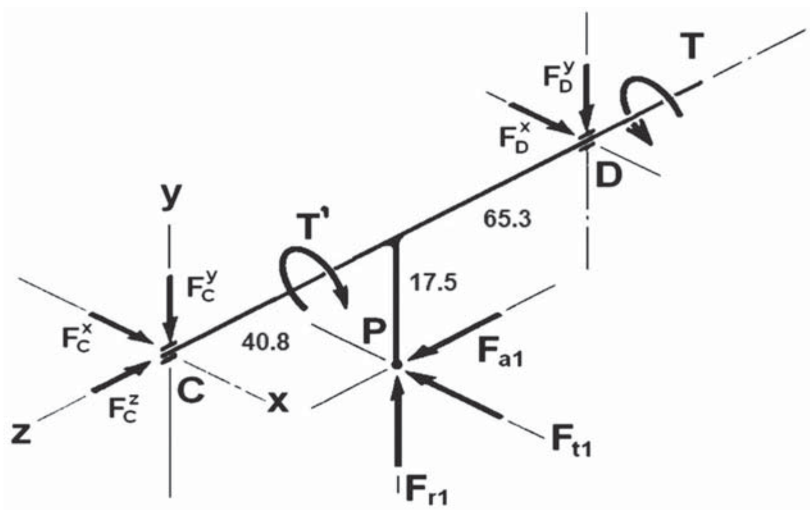


**Fig. 9.** Forces distribution on the worm shaft

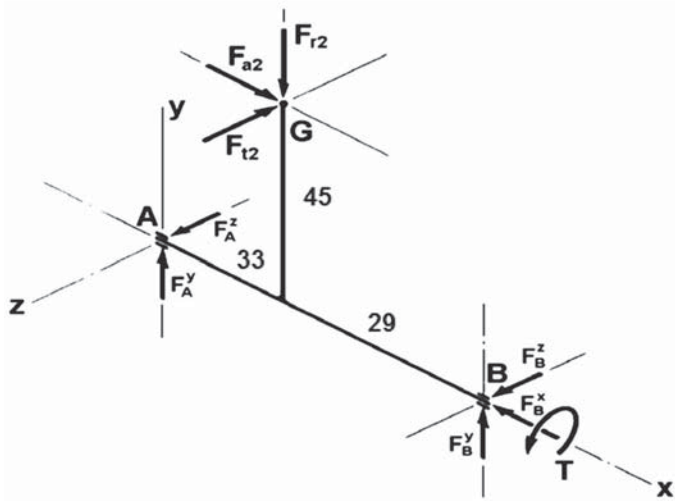


**Fig. 10.** Forces distribution on worm wheel shaft

Based on the above diagrams, the resultant forces acting in every bearing in worm gear have been obtained as functions of input torque and friction coefficient between worm and worm wheel.

Total forces acting in every bearing are the input to the mathematical model of power losses. In the analysis presented in this paper, SKF model has been used (SKF Bearing Calculator).

$$F_A = \sqrt{(F_A^y)^2 + (F_A^z)^2} = \sqrt{\left(\left(\frac{0.0548}{0.00564 + 0.0166 \cdot \mu} - 41.45\right) \cdot T\right)^2 + \left(\frac{0.468 \cdot T \cdot (0.939 - 0.324 \cdot \mu)}{0.00564 + 0.0166 \cdot \mu}\right)^2} \tag{4}$$

$$F_B = \sqrt{(F_B^y)^2 + (F_B^z)^2} = \sqrt{\left(T \cdot \left(\frac{0.0622}{0.00564 + 0.0166 \cdot \mu} + 41.45\right)\right)^2 + \left(\frac{0.532 \cdot T \cdot (0.939 - 0.324 \cdot \mu)}{0.00564 + 0.0166 \cdot \mu}\right)^2} \tag{5}$$

$$F_C = \sqrt{(F_C^x)^2 + (F_C^y)^2} = \sqrt{(35.169 \cdot T)^2 + \left(\frac{T \cdot (0.024 - 0.00567 \cdot \mu)}{0.000598 + 0.00176 \cdot \mu}\right)^2} \tag{6}$$

$$F_D = \sqrt{(F_D^x)^2 + (F_D^y)^2} = \sqrt{(21.947 \cdot T)^2 + \left(\frac{0.117 \cdot T}{0.00564 + 0.0166 \cdot \mu} - \frac{T \cdot (0.024 - 0.00567 \cdot \mu)}{0.000598 + 0.00176 \cdot \mu}\right)^2} \tag{7}$$

### 3.5. Model correlation

The main goal of correlation process was to establish the values of coefficients and constant present in equation 3 for calculation of churning losses and then friction coefficient for calculation of tooth engagement losses. Since simultaneous correlation is very difficult, it was decided to conduct entire correlation process in two steps. As it was already mentioned in paragraph 3.3 churning losses are assumed not to be load dependant and therefor the values of coefficients and constant present in equation 3 will be determined in idle conditions. In order to achieve satisfactory results an average of 5 measurement runs with different rotational velocity were taken and the sum of errors between simulation and measurement results was the minimization criteria. Results of first correlation step are presented in Table 7.

**Table 7**

Comparison of measured and simulated efficiencies for idle conditions

| Rotational velocity [rpm] | Torque [Nm] | |
|---|---|---|
| | Measured | Simulated |
| 1200 | 0.69 | 0.70 |
| 1500 | 0.71 | 0.71 |
| 2000 | 0.72 | 0.72 |
| 2400 | 0.73 | 0.73 |
| 2800 | 0.73 | 0.74 |

Second correlation step concerns tooth engagement and bearing losses as they are coupled by friction coefficient.

Friction coefficient directly influences the bearing load distribution and therefore influences bearing losses. Results of second correlation step are presented in Table 8.

**Table 8**

Comparison of measured and simulated efficiencies for operational conditions

| Torque [Nm] | Efficiency [-] | | | |
|---|---|---|---|---|
| | Rotational velocity 1500 [rpm] | | Rotational velocity 2000 [rpm] | |
| | Measured | Simulated | Measured | Simulated |
| 5 | 0.48 | 0.45 | 0.49 | 0.46 |
| 10 | 0.61 | 0.62 | 0.62 | 0.62 |
| 15 | 0.70 | 0.70 | 0.71 | 0.70 |
| 20 | 0.72 | 0.74 | 0.72 | 0.74 |
| 25 | 0.75 | 0.76 | 0.75 | 0.76 |
| 30 | 0.77 | 0.78 | 0.77 | 0.78 |
| 40 | 0.81 | 0.80 | 0.81 | 0.80 |
| 50 | 0.84 | 0.84 | 0.84 | 0.84 |
| 60 | 0.86 | 0.87 | 0.87 | 0.87 |
| 70 | 0.89 | 0.90 | 0.90 | 0.90 |
| 80 | 0.91 | 0.91 | 0.92 | 0.91 |
| 90 | 0.92 | 0.93 | 0.93 | 0.94 |

Complete correlated numerical model is able to simulate the overall power loss and therefore the efficiency of analyzed device. The result of simulation of worm gear energetic behavior in 1D environment is shown in Figure 11 as a function of input rotational velocity and output torque.

### 3.6. Model validation

After obtaining a correlated efficiency map of considered worm gear it is necessary to validate whether the model represents the energetic behaviour of the object precisely enough. Since the correlation process has been conducted using measurement data in idle condition and in some part of operational condition, the validation has been done with the remaining sets of measurement data.

**Table 9**

Comparison of measured and simulated efficiencies for operational conditions

| Torque [Nm] | Efficiency [-] | | | |
|---|---|---|---|---|
| | Rotational velocity 1200 [rpm] | | Rotational velocity 2400 [rpm] | |
| | Measured | Simulated | Measured | Simulated |
| 5 | 0.47 | 0.44 | 0.5 | 0.47 |
| 15 | 0.70 | 0.70 | 0.71 | 0.71 |
| 25 | 0.75 | 0.76 | 0.76 | 0.76 |
| 40 | 0.81 | 0.80 | 0.82 | 0.81 |
| 50 | 0.84 | 0.84 | 0.85 | 0.84 |

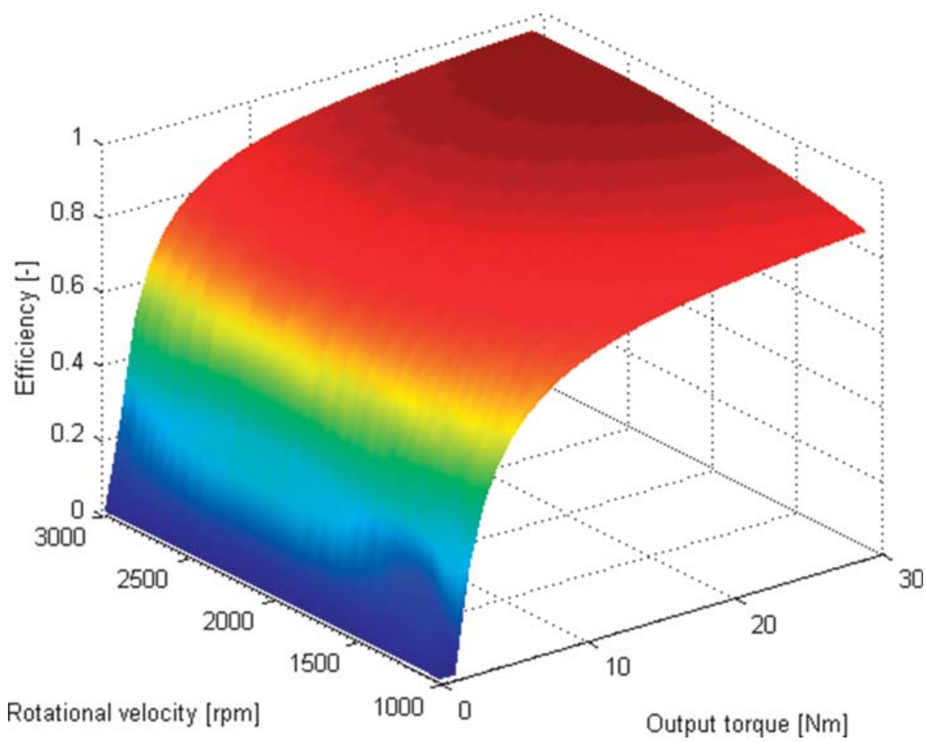


**Fig. 11.** The efficiency of worm gear as a function of output torque and input rotational velocity

Table 9 shows the correspondence between the simulation of correlated model and the measurements results for efficiency values.

## 4. CONCLUSIONS

Experiments conducted on the worm gear provided valuable information on its energetic behaviour. The test data obtained from measurements allowed the determination of losses with respect to different operational conditions (torque, rotational velocity, lubrication). The measurements results have shown that the torque is contributing to the worm gear overall efficiency in the greatest way. Other operational parameters influence the losses insignificantly, however they have been take into account as well. As a result of power losses analysis, it was possible to create a proper model that combines an empirical model based on the performed measurements and a mathematical model based on the considerations contained in the available literature. Model implemented into the AMESim environment has shown a good correspondence between simulated and measured total power loss. Maximum observed error value remained under 7%, therefore worm gear efficiency model proved to be reliable and ready to be used in energy flow simulations and considerations. Since the influence of churning losses is not significant the presented model could be used for energetic considerations on different models of worm gears, however if a high precision is required an additional correlation of churning losses equation parameters should be performed.

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